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Transforming the Non-Linear PDES Governing the Viscous Fluid Flow into Nonlinear ODES and Investigation of Influence of Inclined Magnetic Field and Thermophoresis on Heat and Mass Transfer Wedge

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Abstract: **Nonlinear partial differential equations are transformed into nonlinear ODES using some similarity transformations. From these ODES, the numerical solutions have been obtained using the collocation method, which is in turn implemented in MATLAB software via the bvp4c function. The results of the simulation are presented graphically to depict the effects influence of the above stated parameters on the velocity, temperature and concentration profiles. The results of this study reveal that for this study, fluid velocity is increased by increase in magnetic inclination angle, while concentration and fluid temperature decreases with increase in inclination angle.**

Keywords: **ODE, PDE, Boundary Layer, MHD, inclined angle.**

1. PRELIMINARIES

1.1 NOTATION AND TERMINOLOGY:

f Velocity function, F :Vector with values of f, J :Jacobian , p Fluid pressure, Re Reynolds number, U Free stream velocity, u X-component of velocity, v Y-component of velocity, T Temperature, T_∞ Free stream temperature, T_w Temperature at the surface, U_∞ Free stream velocity f_w Suction or injection, (x, y) Axis direction

1.2 INTRODUCTION:

Fluid flow in a porous medium with mass and heat transfers is of considerable significance from engineering and sciences point of view.(Kandasamy. R., 2005); studied chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects.(Goldsmith and May, 1966); studied the thermophoretic transport involved in simple one-dimensional flows for the measurement of thermophoretic velocity. (Hales. J.M., 1972); studied the thermophoretic deposition in geometry of engineering interest and they solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical flat surface situated adjacent to a large body of an otherwise quiescent air-steam-aerosol mixture. (Derjagun B.V. Rabinorich Ya.I., 1976); Performed various experiments on the thermophoresis of aerosol particles and measured the thermal slip coefficient to calculate thermophoretic velocity, and then compared it with a theoretical one.

2. GOVERNING EQUATIONS

All The equations governing the fluid flows of any kind are based on general laws of conservation of mass, momentum and energy. They are modified to perfectly suit a particular fluid flow. Governing equations are presented and modified subject to the assumption made in order to generate specific equations. In this paper, we consider assumptions made, the general conservation equations of mass and momentum and finally the electromagnetic equations.

$$\left(\vec{J}\times\vec{B}\right)_x \quad \vec{J}=\delta\left(\vec{E}+\vec{V}\times\vec{B}\right) \tag{3.11}$$

$$\Rightarrow\approx\delta\left(\vec{V}\times\vec{B}\right)$$

Where δ is the electrical conductivity coefficient.

$$\vec{B}=\left(-B_o\cos\alpha,-B_o\sin\alpha,0\right)$$

$$\vec{J}=\sigma\left(\vec{E}+\vec{V}\times\vec{B}\right)\approx\sigma\left(\vec{V}\times\vec{B}\right)$$

$$\vec{V}\times\vec{B}=\begin{vmatrix} i & j & k \\ u & 0 & 0 \\ -B_o\cos\alpha & -B_o\sin\alpha & 0 \end{vmatrix}=i(0)-j(0)+k\left(uB_o\sin\alpha-0\right)$$

$$\vec{J}=\sigma uB_o\sin\alpha\widehat{k} \tag{3.12}$$

$$\vec{J}\times\vec{B}=\begin{vmatrix} i & j & k \\ 0 & 0 & \sigma uB\ \sin\alpha \\ -B_0\cos\alpha & -B_0\sin\alpha & 0 \end{vmatrix}=i\left(-\sigma uB_0^2\sin^2\alpha-j\left(\sigma uB_0^2\right)\cos\alpha\sin\alpha\right)+\vec{k}(0)$$

$$\left(\vec{J}\times\vec{B}\right)_x=-\frac{\sigma B_0^2\sin^2\alpha}{\rho}=\frac{\sigma B_0^2}{\rho}\sin^2\alpha\left(u-U\right) \tag{3.13}$$

$$0^0\le\alpha\le 90^0$$

$$\frac{\partial u}{\partial t}+\frac{\partial(u)^2}{\partial x}+\frac{\partial(u,v)}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\sin^2\alpha\left(u-U\right) \tag{3.16}$$

From equation (3.7) we define

$$\frac{DT}{Dt}=\frac{\partial T}{\partial t}+\left(\vec{V}.\nabla\right)T=\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} \tag{3.17}$$

Substituting equation (3.17) into equation (3.7) and bringing in the magnetic inclination term from equation (3.13) we obtain,

$$\frac{\partial T}{\partial t}+\frac{\partial(uT)}{\partial x}+\frac{\partial(vT)}{\partial y}=\frac{1}{\rho Cp}\left(k_f\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho Cp}\left(\frac{\partial u^2}{\partial y}\right)+\frac{\sigma B_0^2}{\rho Cp}\sin^2\alpha\left(u-U\right)^2 \tag{3.18}$$

From equation (3.8), the concentration equation is shown as follows

$$\left(\vec{q}.\nabla\right)C=\left(ui+vi\right).\left(\frac{\partial i}{\partial x}+\frac{\partial j}{\partial y}\right)C \tag{3.19}$$

Similarly, $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ (3.20)

But $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left((D)\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y} \right) \right) \Rightarrow \frac{\partial C}{\partial t} = D\nabla^2$

Similarly $\frac{\partial uC}{\partial x} + \frac{\partial vC}{\partial y} = C\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$ but $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ hence equation 3.19 becomes,

$$\frac{\partial C}{\partial t} + \frac{\partial (uC)}{\partial x} + \frac{\partial (vC)}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(v_T C) \quad (3.21)$$

Introducing the magnetic inclination $(\vec{J} \times \vec{B})_x$ defined in equation (3.13), the equation (3.15) becomes,

$$\frac{\partial u}{\partial t} + \frac{\partial (u)^2}{\partial x} + \frac{\partial (u,v)}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\sin^2 \alpha (u - U) \quad (3.16)$$

From equation (3.7) we define

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (\vec{V}.\nabla)T = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \quad (3.17)$$

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$$\frac{\partial T}{\partial t} + \frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} = \frac{1}{\rho Cp}\left(k_f \frac{\partial T}{\partial y} \right) + \frac{\mu}{\rho Cp}\left(\frac{\partial u^2}{\partial y} \right) + \frac{\sigma B_0^2}{\rho Cp}\sin^2 \alpha (u - U)^2 \quad (3.18)$$

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But $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left((D)\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y} \right) \right) \Rightarrow \frac{\partial C}{\partial t} = D\nabla^2$

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3. MAIN RESULTS

In order to obtain the dimensionless form of the governing equations together with the boundary conditions we introduce the following non-dimensional variables:

$$\eta = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\sigma^{m+1}}}, \psi = \sqrt{\frac{2}{m+1}\frac{vx^{\left(\frac{m+1}{2}\right)}}{\sigma^{\left(\frac{m+1}{2}\right)}}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (4.1)$$

Where η is the similarity variable, ψ is the stream function that satisfies the continuity and is defined by is defined by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ therefore we have

And $u = U(x,t) f'$ (4.2)

$$v = -\sqrt{\frac{2}{m+1} \frac{vx^{\left(\frac{m+1}{2}\right)}}{\sigma^{\left(\frac{m+1}{2}\right)}}} \left(f + \frac{m-1}{m+1} \eta f' \right) \tag{4.3}$$

Where f non-dimensional stream function and prime is denotes differentiation with respect to η. For a variable thermal conductivity is considered as:

$k_f = k_\infty \left(1 + r \frac{T - T_\infty}{T_W - T_\infty} \right)$ Where k_∞ of the ambient fluid and γ is the thermal conductivity variation parameter, Chiam (1998).

Other dimensionless parameters used in the study include,

$\beta = \frac{2m}{m+1}$ Wedge angle, $\Omega = \beta\pi$ Wedge angle parameter, $pr_\infty = \frac{\mu c_p}{k_\infty}$ Prandtl number, S_c Schmidt's number

$N_t = \frac{T_\infty}{T_w - T_\infty}$ Thermophoresis parameter, $N_c = \frac{C_\infty}{C_w - C_\infty}$ Concentration ratio

$Ha = B_0 \sqrt{\frac{\sigma x}{\rho u}}$ Local Hartman number, $E_C = \frac{u}{c_p \left(\frac{T_\infty}{T_w - T_\infty} \right)}$ Eckert number

$f_w = \frac{v_w(x,t)}{\left[\sqrt{\frac{m+1}{2} \frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}} \right]}$ Wall mass transfer coefficient.

Employing the transformations, we obtain the following equations;

Equation (3.16) is transformed to an ODE as follows:

We differentiate the LHS terms of equation (3.16) as shown below

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}\frac{\partial f'}{\partial t} + f'\frac{\partial u}{\partial t} = u\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial t} + f' \cdot \frac{\partial u}{\partial t} u f'' \frac{\partial \eta}{\partial t} + f'\frac{\partial u}{\partial t} \tag{4.4}$$

$$\frac{\partial u^2}{\partial x} = \frac{\partial u^2}{\partial u}\frac{\partial u}{\partial x} = 2u\frac{\partial u}{\partial x} = 2uf'\frac{\partial}{\partial x}[uf'] = 2uf'\left[\frac{u\partial f'}{\partial x} + f'\frac{\partial u}{\partial x}\right] = 2uf'\left[uf''\frac{\partial \eta}{\partial x} + f'\frac{\partial u}{\partial x}\right] \tag{4.5}$$

$$\frac{\partial (uv)}{\partial y} = u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = uf'\frac{\partial v}{\partial y} + v\frac{\partial}{\partial y}(uf') = uf' \left\{ \begin{bmatrix} -\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}} \\ \left(\frac{\partial f}{\partial \eta} + \frac{m-1}{m+1}\frac{\partial \eta f'}{\partial y}\right) \end{bmatrix} + \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}} \left(\frac{\partial f}{\partial y} + \frac{m-1}{m+1}\eta f'\right)\left(uf''\frac{\partial \eta}{\partial y} + f'\frac{\partial u}{\partial y}\right) \right] \right\} \tag{4.6}$$

From equation (3.15) we define

$$\frac{\partial u}{\partial y} = Uf''\frac{\partial \eta}{\partial y} + f'\frac{\partial U}{\partial y} \tag{4.7}$$

$$\frac{\partial u}{\partial t} = vx^m \frac{\partial \delta^{-(m+1)}}{\partial t} = vx^m(-m-1)\delta^{-m-2}\frac{\partial \delta}{\partial t} = \frac{vx^m}{\delta^{(m+1)}}\left(\frac{-m-1}{\delta}\right)\frac{\partial \delta}{\partial t} = u\left(\frac{-m-1}{\delta}\right)\frac{\partial \delta}{\partial t} \tag{4.8}$$

And $\frac{1}{\rho}\frac{\partial p}{\partial x} = u\frac{1}{\rho}\frac{\partial u}{\partial x} = \frac{vx^m}{\sigma^{m+1}}\frac{vx^m}{\sigma^{m+1}}\frac{m}{x}$ (4.9)

Then substituting equations (4.11)-(4.16) into equation (3.16) we get equation (4.10) below

$$\left.\begin{aligned}
&uf''\frac{\partial \eta}{\partial t} + f'\frac{\partial u}{\partial t} + 2uf'\left[uf''\frac{\partial \eta}{\partial x} + f'\frac{\partial u}{\partial x}\right] + uf'\left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\left(\frac{\partial f}{\partial \eta} + \frac{m-1}{m+1}\frac{\partial \eta f'}{\partial y}\right)\right] + \\
&\left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\left(\frac{\partial f}{\partial y} + \frac{m-1}{m+1}\eta f'\right)\left(uf''\frac{\partial \eta}{\partial y}\right)\right] = -\frac{1}{\rho}\frac{vx^m}{\sigma^{m+1}}\frac{vx^m}{\sigma^{m+1}}\frac{m}{x} + \\
&v\left[uf''\frac{\partial^2 \eta}{\partial y^2} + \frac{\partial \eta}{\partial y}uf''\frac{\partial \eta}{\partial y}\right] - \frac{\delta B_0^2}{\rho}(Uf' - u)
\end{aligned}\right\} \tag{4.10}$$

By definition, $U = \frac{vx^m}{\sigma^{m+1}}$ and is common throughout the terms in equation (4.10) and so we divide through to have;

$$\left.\begin{aligned}
&f''\frac{\partial \eta}{\partial t} + f'\left(\frac{m-1}{m+1}\right)\frac{\partial \delta}{\partial t} + 2uff''\frac{\partial \eta}{\partial y} + 2(f')^2\frac{\partial u}{\partial x} + \left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\right] f\frac{\partial \eta}{\partial y}\frac{\partial \eta}{\partial y} + \\
&\left[-\sqrt{\frac{2}{m+1}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}}\left(\frac{m-1}{m+1}\right)\left(ff''\frac{\partial \eta}{\partial y}\right)\right] = -u\frac{1}{\rho}\frac{vx^m}{\sigma^{m+1}}\frac{m}{x} + vf''\frac{\partial^2 \eta}{\partial y^2} + vf'''\left(\frac{\partial \eta}{\partial y}\right)^2 - \\
&\frac{\delta B_0^2}{\rho u}\sin^2\alpha(f' - 1)
\end{aligned}\right\} \tag{4.11}$$

The derivatives in equation (4.11) are obtained by partially differentiating as follows

$$\left.\begin{aligned}
&\frac{\partial \eta}{\partial t} = y\sqrt{\frac{m+1}{2}}\sqrt{x^{m-1}}\frac{\partial}{\partial t}\delta^{\frac{-m-1}{2}} = y\sqrt{\frac{m+1}{2}}\sqrt{x^{m-1}}\left(\frac{-m-1}{2}\right) = \frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial \delta}{\partial t} \\
&\frac{\partial \eta}{\partial x} = y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{\partial}{\partial x}x^{\frac{m-1}{2}} = y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{-m-1}{2}x^{\frac{m-3}{2}} \\
&\frac{\partial u}{\partial x} = \frac{v}{\delta^{m+1}}mx^{m-1} = \frac{vx^m}{\delta^{m+1}}\frac{m}{x} \\
&\frac{\partial \eta}{\partial y} = \sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\delta^{m+1}}}, \frac{\partial^2 \eta}{\partial y^2} = 0
\end{aligned}\right\} \tag{4.12}$$

Similarly,

$$\left.\begin{aligned}&\frac{vx^m}{\delta^{m+1}}\eta\frac{2}{m+1}\frac{\delta^{m+1}}{m+1}vx^{-\frac{1}{2}}=\frac{4\eta}{m+1}x^{-\frac{1}{2}}\\&x^{\frac{m}{2}-\frac{3}{2}}=x^{\frac{m-1}{2}-\frac{1}{2}}\\&2\left[\frac{vx^m}{\delta^{m+1}}\right]y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\left(\frac{-m-1}{2}\right)x^{\frac{m}{2}-\frac{3}{2}},2\left[\frac{vx^m}{\delta^{m+1}}\right]\eta\left(\frac{-m-1}{2}\right)x^{-\frac{1}{2}}=\frac{2vm}{m+1}\\&\frac{vx^m}{\delta^{m+1}}\frac{m}{x}=U\frac{m}{x}\end{aligned}\right\}\tag{4.13}$$

Substituting equations (4.12) and (4.13) in equation (4.11) we have;

$$\left.\begin{aligned}&\frac{\eta^2}{y^2}f'''-v\frac{\eta^2}{y^2}ff''+u\frac{m}{x}\frac{\eta^2}{y^2v}-2u\frac{m}{x}\frac{\eta^2}{y^2v}(f')^2+\frac{2vm}{m+1}v\frac{\eta^2}{y^2}-2v\frac{\eta^2}{y^2}\frac{\delta^{m+1}}{vx^{m-1}}\\&-2v\frac{\eta^2}{y^2}\frac{\delta^{m+1}}{vx^{m-1}}f\,v\frac{\eta^2}{y^2}-\frac{\delta^{m+1}}{vx^{m-1}}f\,v\frac{\eta^2}{y^2}\eta f'-\frac{2v}{m+1}v\frac{\eta^2}{y^2}\delta\frac{B_0^2}{\rho u}\sin^2\alpha(f'-1)^2=0\end{aligned}\right\}\tag{4.14}$$

Since $v\frac{\eta^2}{y^2}$ is common for all terms in equation (4.11) we divide through to have

$$f'''+ff''+\frac{2m}{m+1}-\frac{2m}{m+1}f'^2-\left(\frac{\delta^{m+1}}{vx^m}\right)(2-2f'-\eta f'')-\frac{2}{m+1}Ha^2\sin^2\alpha(f'-1)=0\tag{4.15}$$

But

$$\left.\begin{aligned}&u\frac{m}{x}\div v\frac{\eta^2}{y^2}=u\frac{y^2}{\eta^2}=\frac{2m}{m+1}==\frac{vx^m}{\delta^{m+1}}\frac{2}{m+1}\frac{\delta^{m+1}}{vx^m}\frac{1}{v}\frac{m}{x}=\frac{2x^m}{m+1}\frac{m}{x}=\\&x^m\frac{2}{m+1}\frac{m}{x}\frac{1}{x^{m-1}}=\frac{2x}{m+1}\frac{m}{x}=\frac{2m}{m+1}=\beta\end{aligned}\right\}\tag{4.16}$$

Hence equation (3.16) is transformed to;

$$f'''+ff''+\beta\left((1-f'^2\right)-\left(\frac{\delta^{m+1}}{vx^m}\right)(2-2f'-\eta f'')-\frac{2}{m+1}Ha^2\sin^2\alpha(f'-1)=0\tag{4.17}$$

In equation (3.18) we obtain the following equivalences;

We first defining T as,

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\rightarrow T=\theta(T_w-T_\infty)+T_\infty\tag{4.18}$$

Equation (3.18) is simplified by partially differentiating to obtain the following;

$$\left.\begin{aligned}&\frac{\partial T}{\partial t}=U\frac{\partial}{\partial t}(\theta)(T_w-T_\infty)+T_\infty=(T_w-T_\infty)\frac{\partial\theta}{\partial t}=(T_w-T_\infty)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial t}=\\&(T_w-T_\infty)\left(\frac{-m-1}{2}\right)\theta'\frac{\eta}{\sigma}\frac{\partial\sigma}{\partial t}\end{aligned}\right\}\tag{4.19}$$

$$\left.\begin{aligned}&\frac{\partial(UT)}{\partial x}=U\frac{\partial(T)}{\partial x}+T\frac{\partial(T)}{\partial x}=Uf'\frac{\partial}{\partial x}(\theta((T_w-T_\infty)+)T_\infty)+(\theta(T_w-T_\infty)+T_\infty)\frac{\partial}{\partial x}uf'+\\&uf'(T_w-T_\infty)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}+(\theta(T_w-T_\infty)+T_\infty)\frac{vx^m}{\sigma^{m+1}}f''-\frac{vx^m}{\sigma^{m+1}}\frac{m}{x}f\end{aligned}\right\}\tag{4.20}$$

$$\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\left(\theta\left(T_w-T_\infty\right)+T_\infty\right)\right) \tag{4.21}$$

But from equations (4.19) and (4.20), $u=\frac{vx^m}{\sigma^{m+1}}\ \frac{\partial\eta}{\partial y}=\sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m-1}}{\delta^{m+1}}}\ ,\frac{\partial\theta}{\partial\eta}=\theta'$ and

$$\frac{\partial\eta}{\partial x}=y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{\partial}{\partial x}x^{\frac{m-1}{2}}$$

The first term on the RHS of equation 3.18 becomes

$$\frac{T_w-T_\infty}{\rho c_p}k_\infty\left[\frac{\partial\theta}{\partial y}\frac{\partial}{\partial y}(1+\gamma\theta)+(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2}\right]=\frac{T_w-T_\infty}{\rho c_p}k_\infty\begin{bmatrix}\theta'\frac{\partial\eta}{\partial y}\gamma\theta'\frac{\partial\eta}{\partial y}+\\(1+\gamma\theta)\frac{\partial}{\partial y}\theta'\frac{\partial\eta}{\partial y}\end{bmatrix}= \tag{4.22}$$

$$\frac{T_w-T_\infty}{\rho c_p}k_\infty(\theta')$$

$$\left.\begin{aligned}&\frac{\partial(VT)}{\partial x}=V\frac{\partial(T)}{\partial x}+T\frac{\partial(V)}{\partial x}=V\frac{\partial}{\partial x}\left(\theta\left(T_w-T_\infty\right)+T_\infty\right)+\left(\theta\left(T_w-T_\infty\right)+T_\infty\right)\frac{\partial}{\partial x}(V)\\&=V\left(T_w-T_\infty\right)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}+\left[\left(T_w-T_\infty\right)\theta+T_\infty\right]\frac{\partial}{\partial x}\left[\sqrt{\frac{m+1}{2}}\frac{2}{m+1}vx\delta^{\frac{\frac{m-1}{m+}}{2}}\left(f+\frac{m-1}{m+}\right)\right]\end{aligned}\right\} \tag{4.23}$$

$$\frac{\partial v}{\partial x}=f\left[(-)\sqrt{\frac{m+1}{2}}\frac{2}{m+1}\frac{vx^{\frac{m-3}{2}}}{\sqrt{\delta^{m+1}}}\frac{m+1}{2}\frac{m-1}{2}\right]-\left[\frac{(m-1)\eta}{4}f'\right]\left[\sqrt{\frac{2}{m+1}}\frac{vx^{\frac{m-3}{2}}}{\sqrt{\delta^{m+1}}}(m-1)\right] \tag{4.24}$$

Substituting equations (4.19)-(4.24) into (3.18), we obtain;

$$\left.\begin{aligned}&\left(T_w-T_\infty\right)\frac{\partial\theta}{\partial\eta}\frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t}+Uf'\left(T_w-T_\infty\right)\frac{\partial\theta}{\partial\eta}y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{m-1}{2}x^{\frac{m-3}{2}}-\\&\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sqrt{\delta^{m+1}}}\left(f+\frac{m-1}{m+1}\eta f\right)\left(T_w-T_\infty\right)\frac{\partial\theta}{\partial\eta}\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{m-1}{2}x^{\frac{m-3}{2}}+\\&\left(T_w-T_\infty\right)\theta+T_\infty\frac{\left(T_w-T_\infty\right)}{\rho c_p}k_\infty\left[(\theta')^2\right]\frac{v\eta^2}{y^2}+(1+v\theta)\frac{v\eta^2\theta''}{y^2}+\\&\frac{\mu}{\rho c_p}\left[u^2\right]f''^2\frac{m-1}{2}\frac{x^{m-1}}{\delta^{m+1}}+u\frac{B_0^2}{\rho c_p}(f'-1)\end{aligned}\right\} \tag{4.25}$$

Substituting the following equivalences into equation (4.25)

$$E_c=\frac{U}{C_p\left(T_w-T_\infty\right)}$$

$$pr_\infty=\frac{\mu c_p}{k_\infty},\mu=vp\rightarrow\frac{\mu c_p}{pr_\infty}=k_\infty$$

And dividing the equation by $\frac{(T_w - T_\infty)}{\rho c_p} k_\infty (1+v\theta)\frac{\eta^2}{y^2}$ we obtain the following terms

$$\theta'^2 \times \frac{(T_w - T_\infty)}{\rho c_p} \times \frac{y^2}{(1+v\theta)\eta^2} = \frac{(T_w - T_\infty)}{\rho c_p} \times \frac{y^2}{(1+v\theta)\eta^2} \frac{v\eta^2}{\frac{m+1}{2}\frac{x^{m-1}}{\delta^{m+1}}} = \frac{v}{1+v}\theta'^2 \tag{4.26}$$

$$u\frac{B_0^2}{\rho c_p}\frac{(f'-1)^2}{1+v\theta}\frac{(T_w - T_\infty)}{\rho c_p} k_\infty \frac{y^2}{\eta^2}, k_\infty = \rho c_p \Rightarrow \frac{pr_\infty}{(1+v\theta)} E_c Ha^2 (f'-1)^2 \tag{4.27}$$

$$\frac{\mu}{\rho c_p}\left[u^2\right] f''^2 \frac{m-1}{2}\frac{x^{m-1}}{\delta^{m+1}} \div \frac{(T_w - T_\infty)}{\rho c_p} k_\infty (1+v\theta)\frac{\eta^2}{y^2} = \frac{f''^2}{(1+v\theta)}\frac{\mu c_p}{c_p((T_w - T_\infty))k_\infty}$$
$$= \frac{pr_\infty}{(1+v\theta)} E_c f''^2 \tag{4.28}$$

$$\frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{v\rho c_p}{k_\infty}\eta(\theta'), v\rho = \mu \to \frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{\mu c_p}{k_\infty}\eta(\theta')\frac{v\rho c_p}{k_\infty(1+v\theta)} f(\theta') = \frac{pr_\infty}{(1+v\theta)} f\theta' \tag{4.29}$$

Putting the terms in equations (4.26-4.29), equation (3.18) becomes;

$$\theta'' + \frac{v}{1+v}\theta'^2 + \frac{pr_\infty}{(1+v\theta)} f\theta' + \frac{\delta^m}{vx^{m-1}}\frac{\partial\delta}{\partial t}\frac{\mu c_p}{k_\infty}\eta(\theta') + \frac{pr_\infty}{(1+v\theta)} E_c f''^2 +$$
$$\frac{pr_\infty}{(1+v\theta)} E_c Ha^2 \sin^2\alpha (f'-1)^2 = 0 \tag{4.30}$$

Equation (3.21) is transformed to an ODE term by term as follows:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial t}(\phi(c_w - c_\infty) + c_\infty) = (c_w - c_\infty)\frac{\partial\phi}{\partial t} = (c_w - c_\infty)\frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial t}, but, \frac{\partial\eta}{\partial t} = \frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t} \tag{4.31}$$

$$\frac{\partial C}{\partial t} = (c_w - c_\infty)\frac{\eta}{\delta}\left(\frac{-m-1}{2}\right)\frac{\partial\delta}{\partial t}\phi', \frac{\partial\phi}{\partial\eta} = \phi' \tag{4.32}$$

$$\frac{\partial(uC)}{\partial x} = \frac{u\partial C}{\partial x} + \frac{C\partial(u)}{\partial x} = Uf''(C_w - C_\infty)\phi'\left[y\sqrt{\frac{m+1}{2}}\frac{1}{\sqrt{\delta^{m+1}}}\frac{-m-1}{2}x^{\frac{m-3}{2}}\right] +$$
$$(C_w - C_\infty)\phi + C_\infty\left[\left(uf'' + f'\frac{vx^m}{\delta^{m+1}}\frac{m}{x}\right)\right] \tag{4.33}$$

$$\frac{\partial(vC)}{\partial y} = V\frac{\partial}{\partial y}(\phi(C_w - C_\infty) + C_\infty) + (\phi(C_w - C_\infty) + C_\infty)\frac{\partial v}{\partial y} = v\frac{\partial}{\partial y}(\phi(C_w - C_\infty) + C_\infty) +$$
$$(\phi(C_w - C_\infty) + C_\infty)\left[-\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(\frac{\partial f}{\partial\eta} + \frac{m-1}{m+1}\frac{\partial\eta f}{\partial y}\right)\right] \tag{4.34}$$

$$\frac{\partial(vC)}{\partial y} = v(\phi'(C_w - C_\infty) + C_\infty) + (\phi(C_w - C_\infty) + C_\infty)\left[-\sqrt{\frac{2}{m+1}}\frac{m+1}{2}\frac{vx^{\frac{m-1}{2}}}{\sigma^{\frac{m+1}{2}}}\left(f' + \frac{m-1}{m+1}\frac{\partial\eta^2 f''}{\partial y}\right)\right] \tag{4.35}$$

$$D\frac{\partial^2 C}{\partial y^2} = D\frac{\partial}{\partial y}\left[\frac{\partial}{\partial \eta}\phi(C_w - C_\infty) + C_\infty \frac{\partial \eta}{\partial y}\right] = D\frac{\partial}{\partial \eta}[(\phi'(C_w - C_\infty))]\frac{\eta}{y}\frac{\partial \eta}{\partial y} =$$
$$D\frac{\partial}{\partial \eta}[(\phi'(C_w - C_\infty))]\frac{\eta}{y}\frac{\eta}{y} = D[((C_w - C_\infty))]\phi''\frac{\eta}{y}\frac{\eta}{y}(\phi')(0) = D(C_w - C_\infty)\phi''\frac{\eta^2}{y^2} \quad (4.36)$$

This is the diffusion term

$$\frac{\partial T}{\partial y}(V_T C) = \frac{KV_T C}{T}, \frac{\partial T}{\partial y}\left(\frac{KV_T C}{T}\right) = -KV\frac{\partial}{\partial y}\left[C\frac{\partial T}{\partial y}\right] = -KV\left[\begin{matrix}\frac{\partial T}{\partial y}.\frac{\partial \frac{CT}{T}}{\partial y} + \\ \frac{C}{T}\frac{\partial^2 T}{\partial y^2}\end{matrix}\right] =$$
$$KV\left[\frac{\partial}{\partial \eta}\theta(T_w - T_\infty) + T_\infty \frac{\partial \eta}{\partial y}.\frac{\partial \frac{CT}{T}}{\partial \eta}\frac{\partial \eta}{\partial y} + \frac{C}{T}\frac{\partial}{\partial y}\right] \quad (4.37)$$

But $\frac{\partial \frac{CT}{T}}{\partial \eta} = \frac{T\frac{\partial C}{\partial \eta} - C\frac{\partial T}{\partial \eta}}{T^2}$ therefore equation 4.37 becomes

$$\frac{\partial T}{\partial y}(V_T C) = -KV\left[\frac{\eta^2}{y^2}\theta'(T_w - T_\infty)\frac{T\frac{\partial C}{\partial \eta} - C\frac{\partial T}{\partial \eta}}{T^2} + \frac{C}{T}\frac{\partial}{\partial \eta}\left[\theta'(T_w - T_\infty)\frac{\eta}{y}\right].\frac{\partial \eta}{\partial y}\right] \quad (4.38)$$

$$\frac{\partial \frac{C}{T}}{\partial \eta} = \frac{[\theta(T_w - T_\infty) + T_\infty] - [\phi(C_w - C_\infty) + C_\infty]\phi'[T_w - T_\infty]}{[\theta(T_w - T_\infty) + T_\infty]^2}, \div T_w - T_\infty$$
$$-KV\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w - T_\infty) + T_\infty] - [\phi(C_w - C_\infty) + C_\infty]\phi'[T_w - T_\infty]}{[\theta(T_w - T_\infty) + T_\infty]^2}\right] + \frac{\phi(C_w - C_\infty)}{\theta(T_w - T_\infty)}\phi''\frac{\eta^2}{y^2} \quad (4.39)$$

When we divide the equationby $D(C_w - C_\infty)\frac{\eta^2}{y^2}$;

First term reduces to ϕ'' , second term becomes $\frac{v}{D}f\phi'$ but $\frac{v}{D} = S_c$ hence $S_c\phi'$

The third term becomes

$$KV\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w - T_\infty) + T_\infty][\phi'(C_w - C_\infty) + C_\infty] - \phi[C_w - C_\infty +]C_\infty(\phi')(T_w - T)_\infty}{[\theta(T_w - T_\infty) + T_\infty]^2}\right] +$$
$$\frac{\phi(c_w - c_\infty)}{\theta(T_w - T_\infty)}\phi''\frac{\eta^2}{y^2} \div D(C_w - C_\infty)\frac{\eta^2}{y^2}$$
$$= -Ks_c[\theta']\left[\frac{\eta^2}{y^2}\theta'\frac{[\theta(T_w - T_\infty) + T_\infty][\phi'(C_w - C_\infty) + c_\infty] - \phi[C_w - C_\infty +]C_\infty(\phi')(T_w - T)_\infty}{[\theta(T_w - T_\infty) + T_\infty]^2}\right] \quad (4.40)$$

Isolating the terms in equation (4.40) we obtain,

$$
\left.
\begin{array}{l}
-Ks_c\left[\dfrac{\phi[C_w-C_\infty]}{\theta(T_w-T_\infty)+T_\infty(C_w-C_\infty)}\right]\theta'' \\
-Ks_c\left[\dfrac{\phi\theta''}{\theta(T_w-T_\infty)+T_\infty}\right]+\left[\dfrac{\theta''C_\infty}{\theta(T_w-T_\infty)+T_\infty(C_w-C_\infty)}\right]----a \\
\\
Ks_c\dfrac{[[\theta\phi'\theta'(T_w-T_\infty)+T_\infty]]+T_\infty(c_w-c_\infty)]]}{[\theta(T_w-T_\infty)+T_\infty]^2(c_w-c_\infty)} \\
Ks_c\dfrac{[[\theta\phi'\theta']]}{[\theta(T_w-T_\infty)+T_\infty]^2(C_w-C_\infty)}------b \\
\\
Ks_c\left[\theta\theta'^2\dfrac{((C_w-C_\infty)+C_\infty)(T_w-T_\infty)}{(C_w-C_\infty)[\theta(T_w-T_\infty)+T_\infty]^2}\right]=Ks_c\left[\theta\theta'^2\dfrac{((C_w-C_\infty)+C_\infty)\times(T_w-T_\infty)}{(C_w-C_\infty)\times[\theta(T_w-T_\infty)+T_\infty]^2}\right] \\
Ks_c\theta\theta'^2[1+N_c]\times\dfrac{T_w-T_\infty}{[\theta(T_w-T_\infty)+T_\infty]}\dfrac{1}{[\theta(T_w-T_\infty)+T_\infty]} \\
Ks_c\theta\theta'^2[1+N_c]\times\dfrac{1}{N_t}\dfrac{1}{[\theta(T_w-T_\infty)+T_\infty]}----------c
\end{array}
\right\} \quad (4.41)
$$

Simplifying and rearranging we now obtain

$$\phi''+s_c f\phi''+s_c\lambda\eta\phi+\frac{Ks_c}{N_t+\theta}\left[[(N_c+\phi)]\theta''+\theta'\phi'-\left(\frac{N_c+\phi'}{N_{tc}+\theta}\right)\theta'^2\right]=0 \quad (4.42)$$

The transformed boundary conditions are as follows

$$f=f_w, f'=0, \theta=1, \phi=1 \text{ At } \eta=0 \text{ and } f'=0, \phi=0 \text{ as } \eta\to\infty \quad (4.43)$$

To locally make the equations similar, and using the defined dimensionless parameters, we let, $\dfrac{\delta^m}{vm^{m-1}}\dfrac{d\delta}{dt}=\lambda$ hence the equations (4.17,4.30,4.42) become(4.44,4.4.45,4.46) respectively as shown below

$$f'''+ff''+\beta(1-f'^2)-\left(\frac{\sigma^{m+1}}{vx^{m-1}}\right)(2-2f'-\eta f'')-\frac{2}{m+1}(Ha\sin\alpha)^2(f'-1)=0 \quad (4.44)$$

$$
\left.
\begin{array}{l}
\theta''+\dfrac{v}{1+v}\theta'^2+\dfrac{pr_\infty}{(1+v\theta)}f\theta'+\dfrac{\delta^m}{vx^{m-1}}\dfrac{\partial\delta}{\partial t}\dfrac{\mu c_p}{k_\infty}\eta(\theta')+\dfrac{pr_\infty}{(1+v\theta)}E_c f''^2+ \\
\dfrac{pr_\infty}{(1+v\theta)}E_c(Ha\sin\alpha)^2(f'-1)^2=0
\end{array}
\right\} \quad (4.45)
$$

$$\phi''+s_c f\phi'+s_c\lambda\eta\phi+\frac{Ks_c}{N_t+\theta}\left[[(N_c+\phi)]\theta''+\theta'\phi'-\left(\frac{N_c+\phi'}{N_{tc}+\theta}\right)\theta'^2\right]=0 \quad (4.46)$$

We then reduce the high order equations (4.49-4.51) to first order.

We let,

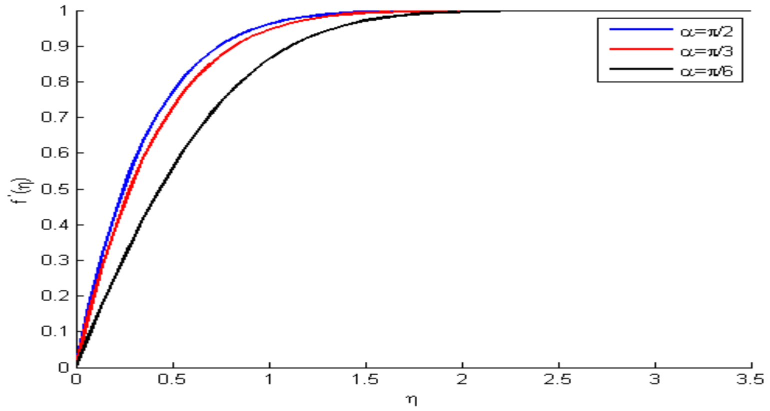
$$\left.\begin{aligned} y_1 &= f \\ y_2 &= f' \\ y_3 &= f'' \\ y_4 &= \theta \\ y_5 &= \theta' \\ y_6 &= \phi \\ y_7 &= \phi' \end{aligned}\right\} \qquad (4.47)$$

$$y_3' = y''' = -y_1 y_3 - \beta\left(1 - y_2^2\right) + \lambda\left(2 - 2y_2 - \eta y_3\right) - \frac{2}{m+1}\left(Ha\sin\alpha\right)^2\left(y_2 - 1\right) = 0 \qquad (4.49)$$

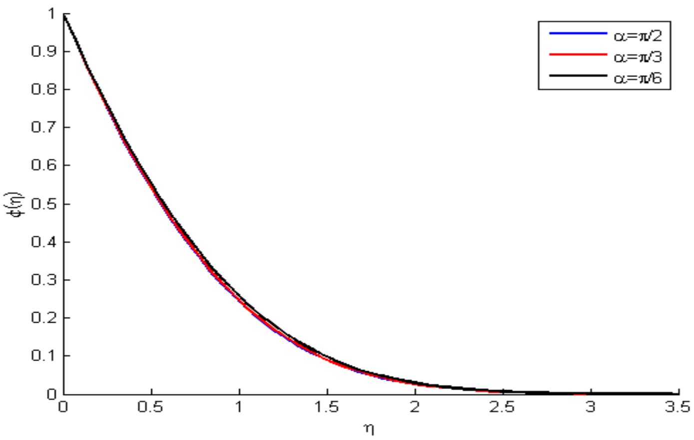
$$\phi'' = y_7^1 = -s_c y y_7 - s_c \lambda \eta y_7 - \frac{Ks_c}{N_t + y_4}\left[\left[\left(N_c + y_6\right)\right]y \ + y_5 y_7 - \left(\frac{N_c + y_6}{N_{tc} + y_4}\right) y_7^2\right] = 0 \qquad (4.50)$$

$$y_5 = \theta'' = \frac{-pr_\gamma y y_5 - \lambda pr_\gamma \eta y_5 - pr_\gamma E_c\left(y_3\right)^2 - pr_\gamma E_c\left(Ha\sin\alpha\right)^2\left(y^2 - 1\right)^2}{1 + \frac{\gamma}{1+\gamma y_3}} \qquad (4.51)$$

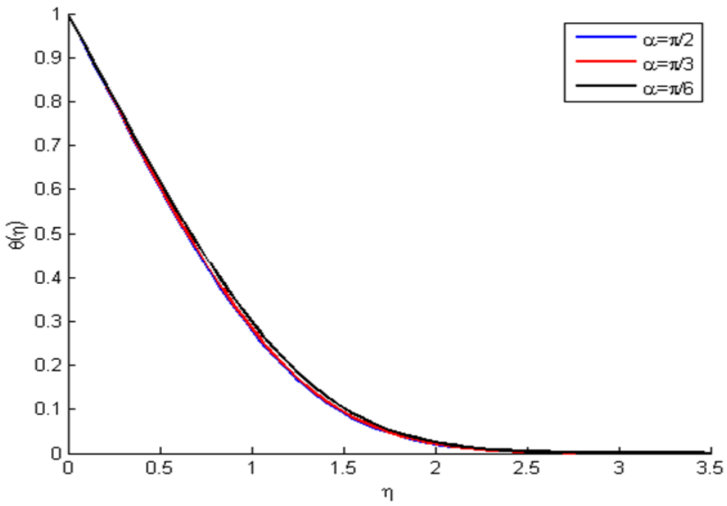
Effects of variation of angle of inclination α



Effect of variation of α on velocity:



Effect of variation of α on concentration:



Effect of variation of α on temperature:

It is clear from figure that increase in inclined angle increases the velocity profiles. It is evident from figures that increase in inclined angle, temperature and the concentration profiles of the fluid decrease. This is because increase in the angle of inclination helps to reduce the thermal and concentration boundary layer thickness whose effect is reduction in temperature and concentration respectively.

Table 1:Computation showing the effect of variation of angle of inclination (α) on skin friction, local Nusselt number and thermophoretic particle deposition V_d

α	cf	Nu	Vd
$\frac{\pi}{2}$	0.1117	19.5410	50.0531
$\frac{\pi}{3}$	0.0996	19.4184	49.7522
$\frac{\pi}{6}$	0.0627	18.9556	48.6499

The increase in angle of inclination increases skin friction, the Nusselt number but reduces thermophoretic deposition.

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